SHORTER ARTICLES AND CORRESPONDENCE

A NOTE ON THE DEGREE OF ACCURACY OF BIOMETRIC CONSTANTS

The statement is frequently made, either in comment or criticism upon biometric work, that such work is often caused to take on an unwarranted appearance of precision and exactness by the keeping of a larger number of decimal places in the tabled constants than the character of the original data justifies. The contention is made that under no circumstances whatsoever can any statistical constant be more accurate than the data on which it is based. It is held that if one makes a series of measurements accurate to a tenth of a millimeter, it is a logical absurdity to table the mean or standard deviation deduced from these measurements to hundredths of a millimeter. Not only is this contention made from time to time by biologists, but occasionally even by a mathematician, a fact which of course tends strongly to confirm the biologist in his opinion. Thus Engberg[1] specifically says (p. 11) referring to mortality statistics: "The constants can not be more accurate than the data on which they are based."[2]

The reply which the statistician makes to the criticism that constants can not be more accurate than the data on which they are based is generally that the accuracy of a statistical constant depends not alone on the accuracy of the original measurements but also upon the number of such measurements. Further it is pointed out that, because of this fact, it is possible to deduce from measurements known to be individually inaccurate constants of a high degree of accuracy, *provided* that the errors in the measurements are unbiased (that is as often in excess as in defect of the true value) and that there are enough of the data. Finally the statistician contends that the only proper measure of the accuracy of a statistical constant

[1] Engberg, C. C. The Degree of Accuracy of Statistical Data. *Univ. of Nebraska Studies,* Vol. III, No. 2, pp. 1–14, 1903.

[2] In passing it may be said that any one who is sufficiently interested in the phenomenon of a professional mathematician taking this curious position will find an entirely adequate and satisfactory discussion of the matter in *Nature.* Vol. 69, p. 93, where Engberg's paper is reviewed.

(always assuming that the original data are not collected in a deliberately dishonest or biased manner) is its "probable error." Unfortunately this statement of the case appears not to carry conviction to the non-statistical worker. It has seemed to the writer that if the assertion made by the statistician regarding the point under discussion is true, it ought to be possible to demonstrate it in such a manner as to carry conviction to anybody.

With this object in view the experiment to be described was tried. Some time ago the writer measured for another purpose the lengths of 450 hens' eggs. The measurements were made with a large steel micrometer caliper manufactured by Browne-Sharpe & Co., reading directly to hundredths of a millimeter. The utmost care was exercised in the making of the measurements; they were all made under the same conditions as to light, temperature, etc.; the caliper was held in a specially constructed stand to get rid of the error arising from expansion and contraction if it is held in the hand; the micrometer screwhead was fitted with a ratchet which mechanically insures that the same pressure shall be exerted on the object in every case; all measurements were made by the same observer who has had considerable experience in close micrometer measuring. The maximum length was the thing measured. There is every reason to believe that these measurements to hundredths of a millimeter are as accurate as it is possible to make them with the instrument used. This being the case all will agree that any statistical constant deduced from them can be held to be accurate to hundredths of a millimeter at least. Now let it be supposed that these eggs had been measured only to the nearest millimeter instead of to the nearest hundredth of a millimeter. By how much would the statistical constants deduced from the "millimeter" data differ from those deduced from the "hundredth millimeter data?"[3]

To answer this question it is necessary to calculate some statistical constant for the two sets of data. The mean was chosen as the simplest possible constant. The actual measurements to hundredths of a millimeter were used as one set of data. The "millimeter" data were obtained by discarding the decimals of the original measurements. In this discarding a record was raised 1 mm. whenever the decimal portion of the original

[3] The biometrician will, of course, recognize that the problem here involved is the same as that of the influence of the fineness of grouping on the value of constants.

figure was .51 or greater. When the decimal part of the record was .49 or less the integral part stood unchanged. In the 450 measurements there were six cases in which the decimal portion of the record was exactly .50. In one half of these cases the record was raised 1 mm. and in the other half was left unchanged, when the decimals were discarded. This is obviously the only fair way of dealing with such cases since, for example, 51.50 is exactly as near to 51 as it is to 52.

The original measurements and the "millimeter" data after discarding the decimals were then each added and re-added with a calculating machine. The resulting sums were:

When the measurements were kept to the nearest hundredth of a mm.	When the measurements were kept to the nearest whole mm.
25,341.95	25,346

Dividing each of these figures by the total number of cases, 450, we get for the means the following:

Mean from "hundredth mm. data"	Mean from "millimeter data"
56.315	56.324

The difference between these two figures is .009. That is, there is no difference between the two averages until the third decimal place is reached. To two places of figures both means are 56.32. But this can only mean that the mean or average obtained when the records are made only to the nearest millimeter is *more* accurate, by two places of decimals, than the data on which it is based.

In interpreting this statement of fact it must not be held to signify that biometric measurements should not be made with the greatest attainable degree of accuracy. Because statistical constants, when the number of cases dealt with is large, are more accurate than the data on which they are based gives no excuse for rough measuring. The reason for this, of course, lies in the principle, which actual experience shows to be correct, that the finer and more accurate the measuring the less chance of the data being unconsciously biased. Statistical constants can only be more accurate than the original data when the data are strictly unbiased. The "applied psychology" of practical measuring teaches that unconscious bias goes out of the records just in proportion as the measurements are made finer.

RAYMOND PEARL.

MAINE EXPERIMENT STATION.